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A practical method to apply hull girder sectional loads to full-ship 3D finite-element models using quadratic programming

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A practical method to apply hull girder sectional loads to full-ship 3D finite-element models using quadratic programming

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Interest in the seakeeping loads of vessels has increased dramatically in recent years. While many studies focused on predicting seakeeping loads, little attention was given on how loads are transferred to 3D finite-element models. In current design practice, methods for predicting seakeeping motions and loads are mainly based on the potential flow theory, either strip theory methods or 3D-panel methods. Methods based on strip theory provide reasonable motion prediction for ships and are computationally efficient. However, the load outputs of strip theories are only hull girder sectional forces and moments, such as vertical bending moment and vertical shear force, which cannot be directly applied to a 3D finite-element structural model. Methods-based 3D panel methods can be applied to a wide range of structures, but are computationally expensive. The seakeeping load outputs of panel methods include not only the global hull girder loads, but also panel pressures, which are well suited for 3D finite-element analysis. However, because the panel-based methods are computationally expensive, meshes used for hydrodynamic analyses are usually coarser than the mesh used for structural finite-element analyses. When pressure loads are mapped from one mesh to another, a small discrepancy at the element level will occur regardless of what interpolation method is used. The integration of those small pressure discrepancies along the whole ship inevitably causes an imbalanced structural finite-element model. To obtain equilibrium of an imbalanced structural model, a common practice is to use the ‘inertia relief’ approach. However, this type of balancing causes a change in the hull girder load distribution, which in turn could cause inaccuracies in an extreme load analysis (ELA) and a spectral fatigue analysis (SFA). This paper presents a practical method to balance the structural model without using inertia relief. The method uses quadratic programming to calculate equivalent nodal forces such that the resulting hull girder sectional loads match those calculated by seakeeping analyses, either by strip theory methods or 3D-panel methods. To validate the method, a 3D panel linear code, MAESTRO-Wave, was used to generate both panel pressures and sectional loads. A model is first loaded by a 3D-panel pressure distribution with a perfect equilibrium. The model is then loaded with only the accelerations and sectional forces and moments. The sectional forces and moments are converted to finite-element nodal forces using the proposed quadratic programming method. For these two load cases, the paper compares the hull girder loads, the hull deflection and the stresses, and the accuracy proves the validity of this new method.

Keywords: quadratic programming; seakeeping sectional load mapping; finite-element model load balance

Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Constraint matrix</td>
</tr>
<tr>
<td>b</td>
<td>Vector of sectional forces and moments</td>
</tr>
<tr>
<td>xc, yc, zc</td>
<td>Center of gravity</td>
</tr>
<tr>
<td>xi, yi, zi</td>
<td>Finite-element nodal location</td>
</tr>
<tr>
<td>fxi, fyi, fzj</td>
<td>Finite-element nodal force</td>
</tr>
<tr>
<td>Fxj</td>
<td>Cross-section surge force</td>
</tr>
<tr>
<td>Fyj</td>
<td>Cross-section vertical shear force</td>
</tr>
<tr>
<td>Fzj</td>
<td>Cross-section horizontal shear force</td>
</tr>
<tr>
<td>Mxj</td>
<td>Cross-section torsional moment</td>
</tr>
<tr>
<td>Myj</td>
<td>Cross-section horizontal bending moment</td>
</tr>
<tr>
<td>Mzj</td>
<td>Cross-section vertical bending moment</td>
</tr>
<tr>
<td>n</td>
<td>Number of nodes in a segment or a full-ship model</td>
</tr>
<tr>
<td>m</td>
<td>Number of sections in a model</td>
</tr>
<tr>
<td>X</td>
<td>Vector of nodal forces</td>
</tr>
</tbody>
</table>

1. Introduction

With a continuous demand for innovative ship designs and more strict requirements from class societies, interest in using seakeeping loads for ship structural design has increased dramatically in recent years. Tools ranging from simple 2D strip theories to complex 3D-CFD numerical simulation methods have been used for practical designs. Most of the seakeeping tools are capable of outputting hull girder loads, such as bending moment and shear force, and some of the tools can also output panel pressure. Examples of the latter are VERES (strip theory), WAMIT, HydroStar and Precal. Other tools such as SMP and ShipMo can only provide hull girder loads, which cannot be directly applied to a 3D finite-element structural model. Panel-based hydrodynamic analysis is well suited for transferring hydrodynamic panel pressure to 3D finite-element structural models.
However, because meshes for hydrodynamic analyses are often much coarser than the corresponding finite-element model meshes, it is necessary to map panel pressure from one mesh to another. Various interpolation methods have been proposed and used in design practice. However, accurately transferring seakeeping panel pressure loads to a finite-element model is still a difficult task. In the hydrodynamic model, solving the equations of motion and integrating the pressure gives a perfect equilibrium, but mapping the pressure to the finite-element model causes imbalanced forces and moments, which must be either removed (rebalancing) or countered by a further set of correcting forces. In the ‘inertia relief’ method, these are fictitious inertial forces that come from modifying the accelerations (ABS 2010a).

Recently, Malenica et al. (2008) proposed a method which maps the panel source strength instead of the panel pressure from a hydrodynamic mesh to the structural mesh, and then formulates the equations of motion in the structural mesh. Because the equations of motion are based on the structural mesh, the method results in a perfectly balanced structural model. However, there are limitations on this method: (A) The method is only applicable to 3D potential flow panel methods. For loads obtained from RANS codes, the direct pressure interpolation is still the only way to transfer loads from one mesh to another. (B) Because the hydrodynamic coefficients are in general not available as an external output and because of the complexity of formulating the equations of motion, the approach can only be materialised within a seakeeping tool. It is not practical for a third party to apply the method for mapping hydrodynamic panel pressure to a structural model. Consequently, in today’s design practice, the approach of using direct pressure mapping is still widely used, and the inertia relief method (ABS 2010b) is used to rebalance the model at the final stage. However, there are two shortcomings to this approach. Firstly, the additional inertial forces cause a change in the hull girder response (such as bending moment) as is shown in the next section. Secondly, the change of the accelerations has to be relatively small to ensure the fictitious inertial forces do not significantly distort the original structural response. This often requires a visual inspection and engineering judgments. For the extreme load analysis (ELA), where the number of load cases is limited, visually inspecting each load case is possible. However, for the spectral fatigue analysis (SFA), where there are thousands of load cases, it is not practical.

In this paper, an approach is proposed to achieve equilibrium based on the sectional forces and moments of the hydrodynamic analysis. The idea of applying correcting nodal forces to a finite-element model is not new. CSR (2010) published a guide on how to manually impose vertical forces on frames (ship sections) to obtain a shear force envelope for a partial model. The bending moment envelope is then achieved by applying moments at the ends of the partial model. This method cannot be used for a full-ship model because the vertical shear force balance is just one of the six equations of equilibrium. The added nodal forces which correct the vertical shear force could cause an imbalance in the bending moment and/or torsional moment. Furthermore, it is not practical to manually adjust nodal forces to match six degrees of freedom equilibrium of a seakeeping analysis.

To solve this problem systematically, the method presented herein uses classical Quadratic Programming (QP) to find a set of equivalent nodal forces for the finite-element model. The mathematical objective is to minimise these equivalent nodal forces, and the constraints are that the six hull girder sectional forces, must match the target values from the hydrodynamic analysis at each station (or section). The equivalent nodal forces can be either complete, where panel pressures and other forces are not present, or additive, where panel pressures and inertia forces are already available. The method is easy to implement and is applicable to strip theory based methods, 3D-panel methods and other CFD methods.

2. Significance of equilibrium for full-ship finite-element analysis

For a floating structure, it is vital to obtain equilibrium before performing a finite-element analysis. An imbalanced model causes an unrealistic result. To illustrate this, a design exercise of a floating full ship is given in this section. A finite-element model was provided by NAPA Ltd of a nominal frigate 150 m long and displacing 4000 tons. The model was created using NAPA-Steel as a moulded form structural model, as shown in Figure 1. The moulded form NAPA model was given a mass distribution and was balanced using NAPA’s internal hydrostatic kernel. The balanced floating condition had a draft of 3.96 m, with no heel and $-0.382^\circ$ trim.

The NAPA-Steel/MAESTRO (MAESTRO 2012) interface program can generate a full-ship finite-element mesh in

Figure 1. A moulded form structural model. (This figure is available in colour online).
MAESTRO format from a moulded form structural model with one click of a button. The generated finite-element model has over 61,000 nodes and 125,000 elements, as shown in Figure 2. This interface program also automatically translates the compartment and wetted surface definitions as MAESTRO groups. In addition, the weight distribution, tank loads and floating condition defined in NAPA hydrostatic module are also translated into MAESTRO. With a complete finite-element mesh and load definition, the generated finite-element model is ready for a linear static analysis without any additional manual editing.

Figures 3 and 4 show the weight and buoyancy distribution of NAPA and MAESTRO, respectively. While the weight distribution of NAPA and MAESTRO has good agreement, the buoyancy distribution between the finite-element model using the initial floating position provided from NAPA and the original NAPA hydrostatic model does not. Such a discrepancy may be caused by several possible reasons: (A) Rudders and propellers are usually not modelled in the finite-element model for hull girder strength analysis. (B) The integration schemes are different. The buoyancy calculation in NAPA is volume based, using continuous curves and/or surfaces. For MAESTRO, the buoyancy is a result of integrating the hydrostatic pressure over the faceted shell elements in the finite-element model.

To solve the previous finite-element model, three nodal constraints were placed near the longitudinal neutral axis of the model to prevent the rigid body motion, with two located at the stern and one at the bow. If the model is properly balanced, the restraining forces will be negligible. In order to check the balance, MAESTRO computes and plots hull girder responses before the restraints are applied. Figure 5 shows the vertical bending moment and vertical shear force. Since the curves are not closed, they reveal that the model is not balanced. Figure 6 shows the resulting bending moment distribution, which includes the reaction forces due to the constraints. Figure 7 shows the deflection and stress distribution. The lack of buoyancy at the stern and the resulting nonnegligible restraint forces cause an incorrect change of sign in the bending moment and in the curvature of the deflected hull.

Finite-element tools specifically developed for floating structures, such as MAESTRO, usually provide a ‘hydrostatic balancing kernel’ by which an imbalanced finite-element model can be automatically rebalanced by adjusting draft, heel and trim. After this hydrostatic balancing, the MAESTRO model has a draft of 4.07 m and a trim of 0.437°. Once the model is balanced, the buoyancy discrepancy

![Figure 2. Finite-element model generated from the NAPA moulded form model. (This figure is available in colour online).](image1)

![Figure 3. Weight distribution comparison. (This figure is available in colour online).](image2)

![Figure 4. Buoyancy distribution comparison. (This figure is available in colour online).](image3)

![Figure 5. Bending moment and shear force distribution before applying restraining forces. (This figure is available in colour online).](image4)
between NAPA and MAESTRO is corrected, as shown in Figure 8. Likewise, the distributions of vertical bending moment and shear force are correctly closed at both ends as shown in Figure 9, and the corresponding deflection and stress distribution are as expected, as shown in Figure 10.

There are two situations in which hydrostatic balancing is not possible: firstly, when using general purpose finite-element programs such as NASTRAN and ANSYS that do not have this feature, and secondly when the loads are derived from a linear seakeeping analysis, where the mean water surface plane is prescribed. There are two main reasons for the latter case: (1) Hydrostatic balancing can only achieve equilibrium in heave, heel and trim, but not in surge, sway and yaw. (2) A hydrostatic rebalance would cause a change of the mean water surface plane, which would require re-running the linear seakeeping analysis.

In these situations, the ‘inertia relief’ method must be used. Figure 11 shows the distributions of vertical bending moment and shear force after using ‘inertia relief’, and Figure 12 shows the corresponding deflection and stress distribution.
Regardless of the method used to balance the model, hydrostatic balance or inertia relief, the hull girder responses may be changed. As shown in Figure 13, the maximum bending moment increases by 11% with hydrostatic balance and by 24% with inertia relief. In the next section, a new approach is proposed to rebalance a model using Quadratic Programming (QP) which preserves all six of the hull girder forces and moments.

3. Method for calculating equivalent nodal forces

The sectional forces and moments given by a seakeeping program are a valuable description and quantification of the loads acting on the ship, and they should be used as a benchmark in generating the loads in the finite-element model. If a seakeeping program gives only sectional loads, or if the pressures it provides do not give equilibrium, some equivalent nodal forces are needed. We have seen that hydrostatic balancing and inertia relief both cause a departure from the sectional loads. It would be better if the equivalent nodal forces are such as to preserve the known sectional loads. It is difficult to know just how to modify the nodal forces of a large finite-element model to get desired resulting sectional forces and moments. But since we are seeking equivalent forces, it is desirable that they be no larger than needed to achieve their purpose. Our proposal is to use quadratic programming, which is a particular type of mathematical optimisation (Fletcher 1987). It minimises a quadratic function of several variables subject to linear constraints on those variables. For the case of balancing a finite-element model, the quadratic function is the magnitude of the equivalent nodal forces, and the linear constraints are that the resulting sectional forces and moments must match the values from the seakeeping analysis. The problem can be set up either as a single problem for the full-ship model, or as a sequence of smaller problems, each corresponding to one segment of the ship. The segments can be defined by the usual 20 stations or by any other number of cross sections, say \( m \). The arrow in Figure 14 shows a typical segment. In each case, the mathematical problem is

\[
\text{minimize } q = \sum_{i=1}^{n} (f_{x_i}^2 + f_{y_i}^2 + f_{z_i}^2),
\]

where \( i \) is the node index, \( n \) is the number of nodes in the problem (full ship or one segment) and \( f_{x_i}, f_{y_i}, \) and \( f_{z_i} \) are the finite-element nodal forces in the problem. The constraints are the \( 6 \times m \) sectional forces and moments, as defined in Equation (2)

\[
\begin{align*}
\sum_{i=1}^{n} f_{x_i} &= F_{xj}, \\
\sum_{i=1}^{n} f_{y_i} &= F_{yj}, \\
\sum_{i=1}^{n} f_{z_i} &= F_{zj}, \\
\sum_{i=1}^{n} (-f_{y_i}(z_i - z_c) + f_{z_i}(y_i - y_c)) &= M_{xj}, \\
\sum_{i=1}^{n} (-f_{z_i}(x_i - x_c) + f_{x_i}(z_i - z_c)) &= M_{yj}, \\
\sum_{i=1}^{n} (-f_{x_i}(y_i - y_c) + f_{y_i}(x_i - x_c)) &= M_{zj}
\end{align*}
\]

where \( j \) is the section index, and \( m \) is the number of the sections of the model. \( m = 1 \) if the problem is set up for a segment. \( x_c, y_c, \) and \( z_c \) are the coordinates of the centre of gravity, and \( x_i, y_i, \) and \( z_i \) are the finite-element nodal coordinates. \( F_{xj}, F_{yj}, \) and \( F_{zj} \) are the cross-sectional forces, and \( M_{xj}, M_{yj}, \) and \( M_{zj} \) are the cross-sectional moments. Equations (1) and (2) can be rewritten as

\[
\text{minimize } : q(X) = \frac{1}{2} X^T GX
\]

subject to : \( AX = b \),

where \( G = 2I \), \( X \) is a vector of the finite-element model’s nodal forces, \( A \) is the matrix of the linear equality constraint coefficient and \( b \) is a vector of sectional forces and moments. The quadratic programming problem can be solved by the method of Lagrangian multipliers. The Lagrangian function for this problem is

\[
L(X, \lambda) = \frac{1}{2} X^T GX - \lambda^T (A^T X - b)
\]
and the stationary point condition yields the equations

\[
\begin{bmatrix}
G & -A \\
-A^T & 0
\end{bmatrix}
\begin{bmatrix}
X \\
\lambda
\end{bmatrix} = -\begin{bmatrix}
0 \\
b
\end{bmatrix}.
\]

The number of variables in Equation (5) is \( n \times 3 + 6 \times m \). If the problem is set up for solving a segment, then \( m \) is 1, and the problem must be formulated and solved \( m \) times. Although the total number of variables is the same, the computational effort is much less. If the seakeeping program has provided the inertial forces on all the nodes and it is only the hydrodynamic pressure forces that are needed, then the QP problem need only include the wetted nodes.

The basic steps of the QP method are given in Table 1.

4. Numerical validation

We now present two examples to compare the balancing methods and validate the QP method. In the first example, we use the model described in Section 2 (with the incorrect buoyancy) to perform four types of balancing. The first two cases use the QP method with the NAPA bending moment distribution as the target. In the first case, the balancing begins with the original weight and buoyancy distribution. The equivalent nodal forces are calculated such that the resulting bending moment matches the original NAPA bending moment, as shown in Figure 15. In the second case, the QP balancing begins with none of the original weight and buoyancy distribution. In this case, the ‘equivalent’ nodal forces are the total nodal forces. Again, the bending moment matches the target, as shown in Figure 15. A static finite-element analysis was performed for both cases. The hull girder deflections and longitudinal stress distributions are shown in Figures 16 and 17. A comparison of the longitudinal stress distribution in the main deck is given in Table 2.

For comparison, Figure 15 and Table 2 include the results from Figure 13: inertia relief and hydrostatic balancing. In all four cases, the stress pattern is very similar, although the load detail of each load case is different. The peak stresses in the inertia relief case and the hydrostatic balance case are larger because they are the results of the larger corresponding hull girder bending moments, as shown in Figure 15.

The second example is to match section loads derived from a linear seakeeping analysis. The load response of an oblique ocean wave, where longitudinal torsional moment and horizontal bending moment are not negligible, is selected to demonstrate the flexibility of the method. The model is assumed to have a forward speed of 20 knots at a heading of 135° on a unit wave with a wave period of 10 s. A hydrodynamic analysis was performed by MAESTRO-Wave, a frequency-domain potential flow linear 3D panel code. The load output of MAESTRO-Wave included both panel pressures and sectional loads. Because the equations of motion of MAESTRO-Wave are formulated based on the structural mesh, the pressure applied to the finite-element model, along with the inertial force, result in perfect
Table 2. Comparison of the main deck longitudinal stress distribution.

<table>
<thead>
<tr>
<th>Main deck longitudinal stress</th>
<th>$\sigma_x$ (MPA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QP balancing starting with the NAPA weight and buoyancy</td>
<td>92.1</td>
</tr>
<tr>
<td>QP balancing starting without the NAPA weight and buoyancy</td>
<td>90.9</td>
</tr>
<tr>
<td>Using inertia relief balancing</td>
<td>117.9</td>
</tr>
<tr>
<td>Using hydrostatic balancing</td>
<td>104.3</td>
</tr>
</tbody>
</table>

equilibrium. The dynamic pressure distribution of the unit wave with a phase angle 0 is shown in Figure 18.

In the first load case, hydrodynamic panel pressure and the inertial forces are applied to the finite-element model, and there is no need for any correction because it is already a balanced model. Then, the vertical bending moment, vertical shear force, longitudinal torsional moment, horizontal bending moment and horizontal shear force are computed based on the panel pressure integration. In this load case, the individual load components, panel pressures and inertial forces are derived from first principles, so it serves as a benchmark for the following load case.

Figure 18. Hydrodynamic pressure distribution. (This figure is available in colour online).

Figure 19. Vertical bending moment. (This figure is available in colour online).

Figure 20. Vertical shear force. (This figure is available in colour online).
For the second load case, the inertial load was kept intact, but the hydrodynamic panel pressure was not used. The sectional loads calculated from MAESTRO-Wave were used as target values for the QP method to find the ‘optimal’ equivalent nodal forces which were equivalent to the panel pressures. Because the wave-induced panel pressures occur on the external shell, the QP method only needed to calculate equivalent nodal forces at the wetted nodes of the finite-element model. The sectional forces and moments were then recalculated using the inertial forces from MAESTRO-Wave and the equivalent nodal forces from the quadratic programming.

For hull girder sectional loads, Figures 19–23 compare the ‘exact’ solution (using panel pressures with a perfect balance) and the solution without the pressures and using QP to balance the model. In all cases, it is clear that the QP method achieves its purpose. The deflections and stress distributions of the full ship and the main deck, with panel pressure loading and sectional force loading, are shown in Figures 24 and 25.

5. Concluding remarks

This paper presents a practical method for balancing the loads in a finite-element model while matching the hull girder sectional forces and moments obtained from a seakeeping analysis. The method is flexible and easy to implement. One of the main applications of the method is to transfer seakeeping loads, obtained by either strip theory methods or 3D-panel methods, to a finite-element model. Using the method, the nodal force adjustment can be applied at different levels. For example, the method can be used either with or without inertial loads, and with or without panel pressure. With the assistance of this method,
the pressure mapping from a hydrodynamic mesh to a structural mesh does not have to be very accurate. The method is validated by numerical results.

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References


